Seiberg-Witten description of the deconstructed 6D (0,2) theory

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It has recently been suggested that, in the large N limit, a particular four dimensional gauge theory is indistinguishable from six dimensional conformal field theory with (0,2) supersymmetry compactified on a torus. We give further evidence for this correspondence by studying the Seiberg-Witten curve for the "deconstructed" theory and demonstrating that along the reduced Coulomb branch of moduli space (at the intersection of the Higgs and Coulomb branches) it describes the low energy physics on a stack of M5-branes on a torus, which is the (0,2) theory on a torus as claimed. The M-theory construction helps to clarify the enhancement of supersymmetry in the deconstructed theory at low energies, and demonstrates its stability to radiative and instanton corrections. We demonstrate the role of the theta vacuum in the deconstructed theory. We point out that, by varying the theta parameters and gauge couplings in the deconstructed theory, the complex structure of the torus can be chosen arbitrarily, and the torus is not metrically $S^1 \times S^1$ in general.

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I. INTRODUCTION

It has recently been demonstrated that certain four dimensional gauge theories can reproduce the physics of higher dimensional gauge theories, typically up to some energy scale [1]. The four dimensional theory is referred to as the "deconstructed," or "latticized," theory. Gauge theories in more than four dimensions are typically not renormalizable, so the deconstructed theory provides an ultraviolet completion for the higher dimensional theory. The basic idea is to begin with a four dimensional action which, after certain elementary or composite fields get vacuum expectation values (VEVs), is the action for the higher dimensional theory latticized in all but four dimensions, plus corrections which become small if the lattice spacing is relatively small. The lattice spacing can generally not be taken to zero in a controlled way because this requires the infinite coupling limit of the gauge theory. Deconstruction of extra dimensions has proved useful in model building [2–12], and often the number of lattice sites can be taken as small as two while maintaining the relevant features of the extra dimensions.

Deconstruction of supersymmetric (SUSY) models is especially interesting [2–5]. The deconstructed SUSY theory typically has less supersymmetry at high energies than at the low energies for which it describes the higher dimensional model. This unusual behavior is due to an accidental symmetry on the chosen branch of moduli space of the theory, and it must be checked that there are no radiative or nonperturbative corrections that would violate the enhancement of SUSY. In the case where the 5D SUSY gauge theory with eight supercharges is deconstructed to a theory with $\mathcal{N}=1$ SUSY in 4D (four supercharges) [2], stability of the spectrum and other holomorphic data follows from a nonanomalous $U(1)_R$ symmetry which is preserved, or alternatively by the $\mathcal{N}=1$ nonrenormalization theorem, which prevents the generation of $\mathcal{N}=2$ SUSY breaking terms in the superpotential. Here we will study a six dimensional theory with 16 supercharges, deconstructed to a theory with eight supercharges. In this case we will explicitly understand the stability of SUSY enhancement from the Seiberg-Witten description of the theory. This model was studied in [3], and we will generalize the model to the case with arbitrary theta vacuum in the deconstructed theory. For related work, see [6]. By varying the theta vacuum we deconstruct the 6D theory with (0,2) SUSY compactified on a torus with arbitrary complex structure.

The model is a 4D $SU(M)^N$ gauge theory with $\mathcal{N}=2$ SUSY and N bifundamental hypermultiplets Q_i as follows:

	$SU(M)_1$	$SU(M)_2$	$SU(M)_3$	$\cdots$	$SU(M)_{N-1}$	$SU(M)_N$
Q_1	$\square$	$\bar{\square}$	1	$\cdots$	$\cdots$	1
Q_2	1	$\square$	$\bar{\square}$	$\cdots$	$\cdots$	1
	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\ddots$	$\vdots$
Q_{N-1}	1	$\cdots$	$\cdots$	$\cdots$	$\square$	$\bar{\square}$
Q_N	$\bar{\square}$	1	$\cdots$	$\cdots$	1	$\square$

(1.1)

Q_i represents a vectorlike pair of $\mathcal{N}=1$ chiral multiplets in Eq. (1.1). The $SU(M)$ gauge couplings g are all taken to be equal, and we will usually assume that the theta parameters for individual $SU(M)$ gauge group factors are equal. This theory has a Higgs branch, for which the scalars in the Q_i hypermultiplets get VEVs, and a Coulomb branch, for which

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the adjoint scalars in the $\mathcal{N}=2$ vector multiplets get VEVs and the gauge group is generically broken to $\mathrm{U}(1)^{N(M-1)}$. If the Q_i all have equal VEVs $v\mathbf{1}$ (and the adjoints do not have VEVs), then the gauge group is broken to a diagonal $\mathrm{SU}(M)_D$. At energies much smaller than gv/N the theory is 4D $\mathcal{N}=4$ SU(M) gauge theory. If the theta parameters vanish and $N \gg 1$ with g^2/N and gv/N held fixed, then it was argued in [3] that the low lying dyonic spectrum is the Kaluza-Klein (KK) spectrum of 6D $(0,2)_M$ superconformal field theory on a torus of radii $N/(2\pi gv)$ and $g/(8\pi^2 v)$. The correspondence was made precise by starting with a type IIB string theory description of the gauge theory and showing that the corresponding M-theory description on a baryonic Higgs branch (i.e., when the "baryons" Q_i^M have expectation values, in analogy with the baryonic branch in SUSY QCD [13]) is that of a stack of M M5-branes wrapped on the torus, with the 11D Planck length l_P taken to zero. This is the $(0,2)_M$ theory [14] compactified on a torus. In the case $M\to\infty$ the deconstructed theory is a holographic description of type IIB string theory on a *pp*-wave background [15].

We will study the correspondence between the (0,2) theory and its deconstructed version by starting with the type IIA description of the deconstructed theory, and raising it to M theory before moving along the baryonic Higgs branch. This allows us to easily determine the complex and metric structure of the torus in terms of the parameters of the deconstructed theory. The M-theory description involves an M5-brane wrapped on the Seiberg-Witten curve which describes the dynamics of the gauge theory on the Coulomb branch. The Seiberg-Witten description of an $\mathcal{N}=1$ version of this $\mathcal{N}=2$ theory was studied in [16]. The Seiberg-Witten curve in the $\mathcal{N}=2$ case studied here will be useful despite the fact that the relevant branch of moduli space for deconstruction is the Higgs branch, not the Coulomb branch.

There is a baryonic reduced Coulomb branch in which both the Q_i and adjoints have VEVs. The gauge group in the low energy theory along this branch is $\mathrm{U}(1)^{M-1}$. The Coulomb and reduced Coulomb branches meet when the bifundamental VEV v vanishes and the adjoint scalar VEVs among the N gauge group factors are equal. Along the reduced Coulomb branch, the Coulomb and Higgs sectors have a product structure. In other words, Q_i VEVs do not affect the low energy behavior of the adjoint fields, and vice versa. This follows by studying constraints on the Kähler potential from $\mathcal{N}=2$ supersymmetry [13].

The Seiberg-Witten curve becomes singular and factorizes on the intersection of the Higgs and Coulomb branches [17,18]. One factor precisely describes the complex structure of the torus on which the M5-branes are wrapped, which allows us to easily identify the torus in terms of the gauge theory parameters. By the decoupling of the Higgs and Coulomb branches, the complex structure of the torus does not depend on the Q_i VEVs. This is true even along the root of the reduced Coulomb branch, when the adjoint VEVs vanish and the low energy theory has an SU(M) gauge symmetry. This is the domain of moduli space which describes the $(0,2)_M$ theory. Hence, the Seiberg-Witten curve contains information about the $(0,2)_M$ theory on the torus. The low energy dynamics of the (0,2) theory on a torus, which is 4D $\mathcal{N}=4$ SU(M) gauge theory, is recovered exactly without corrections from the additional moduli involving the Q_i hypermultiplet VEVs. The factorization of the Seiberg-Witten curve is easy to understand from the M-theory five-brane description of the theory. It also demonstrates that the enhancement of SUSY at low energies is stable to radiative and instanton corrections. A more detailed analysis is the purpose of this paper.

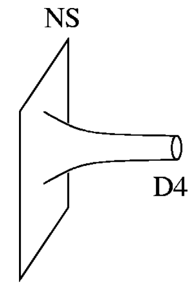

FIG. 1. The intersection of a type IIA D4-brane and NS5-brane is generically smoothed out into a single M5-brane in M theory.

II. TYPE IIA AND M-THEORY CONSTRUCTION OF THE DECONSTRUCTED (0,2) THEORY

Witten demonstrated that the low energy dynamics of a class of $\mathcal{N}=2$ supersymmetric gauge theories, which arise as the low energy theories on configurations of Neveu-Schwarz 5-branes (NS5-branes), and D4-branes of type IIA string theory, can be understood by lifting the corresponding brane configurations to M theory [19]. Intersections of *D*-branes and NS5-branes are generally smoothed out into a single M5-brane, as in Fig. 1. In this interpretation, the D4-brane corresponds to the M5-brane wrapped on the M-theory circle of radius R_{10}, and the NS5-brane corresponds to the M5-brane at a point on the M-theory circle. (The fact that in certain gauge theories strong coupling implies that an M-theory dimension effectively opens up was made use of in a similar context in [20].) The algebraic curve which describes the geometry of the M5-brane was shown in [19] to be precisely the Seiberg-Witten curve [21] from which the holomorphic gauge coupling $\tau=4\pi i/g^2+\theta/2\pi$ and Kähler potential are determined along the Coulomb branch of the corresponding gauge theory.

The 4D theory proposed in [3] as a deconstructed version of the six dimensional (0,2) theory is derived from a type IIA brane configuration as in Fig. 2a. There are N NS5-branes which live in the $(x_0,\ldots,x_3;x_4,x_5)$ directions and M D4-branes stretched along the $(x_0,\ldots,x_3;x_6)$ directions between neighboring pairs of NS5-branes. The x_6 direction is compactified on a circle of radius R_6, which will be identified by the KK spectrum. All of the branes are at $x_7=x_8=x_9=0$ to begin with. The type IIA description is T dual to the type IIB A_{N-1} quiver theory described, for example, in [22]. The gauge theory that lives on the brane configuration described above, at energies below the string scale l_s^{-1}, is an $\mathcal{N}=2$ $\mathrm{SU}(M)^N$ gauge theory with bifundamental hypermultip-

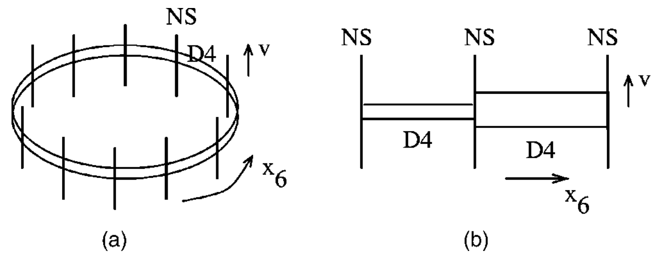

FIG. 2. Type IIA brane configurations for theories studied here. (a) Deconstructed (0,2) theory. There are N NS5-branes and M D4-branes between neighboring NS5-branes. (b) $\mathcal{N}=2$ SU(M) $\times$SU(M) gauge theory with bifundamental hypermultiplet. There are three NS5-branes and M D4-branes between neighboring NS5-branes.

lets Q_i as given in Eq. (1.1). This is the theory described in the Introduction. In fact, there is an additional U(1) in the theory, but nothing is charged under it so it decouples from the theory we are interested in. The gauge coupling of the ith group factor is given by

$$\frac{4\pi}{g_i^2}=\frac{x_6^i-x_6^{i-1}}{g_s l_s}, \tag{2.1}$$

where g_s is the string coupling, and l_s is the string length. The branch of moduli space relevant to the deconstructed (0,2) theory is along the Higgs branch with all the Q_i VEVs equal and proportional to the identity, $Q_i=v\mathbf{1}$. On this branch the gauge group is broken to a diagonal SU$(M)_D$ with a massless adjoint hypermultiplet. The adjoint scalar in the SU$(M)_D$ vector multiplet can still get a VEV in the Cartan subalgebra of SU$(M)_D$, which would generically break the gauge group further to U$(1)^{M-1}$. If the adjoint gets a generic VEV this is a reduced Coulomb branch, an analog of the baryonic branch in [13]. The positions of the D4-branes in the $z=x_4+ix_5$ direction correspond to the VEVs of the adjoint scalars in the SU$(M)^N$ vector multiplet (up to instanton corrections). If the adjoint VEVs are equal for all of the SU(M) gauge group factors, then the D4-branes wrap the circle and decouple from the NS5-branes, so that the NS5-branes can be removed while remaining on the vacuum manifold. (This will be made explicit in the next section.) This is the intersection of the Coulomb and reduced Coulomb branches. The bifundamental hypermultiplets correspond to strings that cross the NS5-branes, and they obtain VEVs when the NS5-branes are removed from the circle in the x_7,x_8,x_9 directions. When all of the NS5-branes are removed from the D4-brane circle, the gauge theory on the circle is generically broken to U$(1)^{M-1}$. This is the reduced Coulomb branch discussed earlier.

The relevant limit of the deconstructed theory is $l_s\to 0$ (in order to decouple the string modes so that the brane dynamics is described by the field theory), $N\to\infty$, $g\to\infty$, $v\to\infty$, $g_s=g/(8\pi^2 v l_s)\to\infty$ such that $N/gv=$const, $g_s l_s=$const. These limits imply that the theory is better described within M theory compactified on a circle $R_{10}=g_s l_s$, which is held fixed in the deconstruction limit. When lifted to M theory the brane configuration is generically described by a single M5-brane wrapped on the Seiberg-Witten curve. The nontrivial geometry of the M5-brane is confined to the directions x_4,x_5,x_6 and the M-theory direction x_{10}. The effective four dimensional low energy theory lives in the directions $x_0,\ldots,x_3$. The D4-branes become M5-branes which are wrapped around both the M-theory circle in the x_{10} direction and the circle in the x_6 direction. For small l_s and constant R_{10}, the 11D Planck length $l_P^3=l_s^2R_{10}$ is small, and the theory which lives on this brane configuration is the 6D (0,2) theory wrapped on a torus up to energies that probe the NS5-branes. We will discuss the relevant energy scales in more detail later. At energies smaller than the inverse size of the torus, the theory is simply 4D $\mathcal{N}=4$ Yang-Mills theory. This demonstration was another approach to the same M5-brane configuration described in [3]. But, as we will see by studying the Seiberg-Witten curve for this theory, the choice of theta vacuum of the deconstructed theory is reflected in the complex structure of the manifold on which the M5-brane is wrapped.

III. FACTORIZATION OF THE SEIBERG-WITTEN CURVE AND ENHANCEMENT OF SUPERSYMMETRY

In this section we will demonstrate the factorization of the Seiberg-Witten curve on the reduced Coulomb branch, and thus the enhancement of supersymmetry. To begin we will avoid the additional complications that arise from putting the x_6 direction on a circle, and first demonstrate in more detail the decoupling of the Coulomb and Higgs branches in the Seiberg-Witten curve for the following $\mathcal{N}=2$ theory with hypermultiplet Q:

	SU$(M)_1$	SU$(M)_2$
Q	$\square$	$\bar{\square}$

Recall that the hypermultiplet is vectorlike, so this theory is anomaly-free. It is described by the brane configuration shown in Fig. 2b. Note that this theory differs from that studied in [23,24] in that the latter had a vanishing superpotential, whereas the $\mathcal{N}=2$ theory studied here has the superpotential

$$W=Q_A^\alpha\phi_\alpha^\beta\tilde{Q}_\beta^A-\tilde{Q}_\alpha^A\varphi_A^B Q_B^\alpha+\lambda\phi_\alpha^\alpha+\mu\varphi_A^A, \tag{3.1}$$

where Q and $\tilde{Q}$ are the two $\mathcal{N}=1$ chiral multiplets in the bifundamental hypermultiplet, and ϕ and φ are the adjoint chiral multiplets of the two SU(M) gauge groups. Greek and roman indices run from 1 to M and refer to the two gauge group factors. The Lagrange multipliers λ and μ enforce the tracelessness of the adjoint fields. Although the superpotential was absent in [24], their analysis of the D-flatness constraints on Q and $\tilde{Q}$ is unchanged as long as the adjoints ϕ and φ do not get VEVs. It was found there that the general solution to the D-flatness constraints was that Q and $\tilde{Q}$ take the forms

$$Q=\begin{pmatrix} q_1 & & \\ & \ddots & \\ & & q_N \end{pmatrix}, \quad \tilde{Q}$$
$$=\begin{pmatrix} \sqrt{q_1^2+c} & & \\ & \ddots & \\ & & \sqrt{q_N^2+c} \end{pmatrix} Ce^{i\beta}, \qquad (3.2)$$

where C is a diagonal SU(M) matrix.[1]

The F-flatness constraints can be read off the superpotential (3.1), namely,

$$\lambda\,\delta^{\alpha}_{\beta}+\tilde{Q}^{A}_{\beta}Q^{\alpha}_{A}=0, \qquad (3.3)$$

$$Q^{\alpha}_{A}\phi^{\beta}_{\alpha}-\varphi^{B}_{A}Q^{\beta}_{B}=0, \qquad (3.4)$$

and two similar equations of motion with respect to φ and Q. Note that generically the bifundamentals Q and $\tilde{Q}$ cannot get VEVs if the adjoints ϕ and φ do have VEVs, but if ϕ and φ have equal nonvanishing VEVs in the Cartan subalgebra of the corresponding SU(M), then Q and $\tilde{Q}$ can get VEVs proportional to the identity. In particular, we can choose $Q=\tilde{Q}=v\mathbf{1}$. This is the reduced Coulomb branch discussed earlier.

We can easily recover this behavior from the Seiberg-Witten curve for this theory. The Seiberg-Witten curve and one-form determine the holomorphic gauge coupling function and Kähler potential on the Coulomb branch as a function of the Coulomb branch moduli of the theory, namely, the VEVs of gauge invariant operators [25]

$$u^{(1)}_{n}=\langle \mathrm{Tr}\ \ \phi^{n}\rangle, \qquad (3.5)$$

$$u^{(2)}_{n}=\langle \mathrm{Tr}\ \ \varphi^{n}\rangle, \qquad (3.6)$$

where $n=2,...,M$. But we will restrict the VEVs to all be equal so that we intersect the Higgs branch, in which case there are only $M-1$ independent moduli in the curve, as opposed to $N(M-1)$.[2] If we define the complex parameters $z=x_4+ix_5$ and $t=\exp(x_6+ix_{10}/R_{10})$ (which is holomorphic because $x_{10}\sim x_{10}+2\pi R_{10}$), then Witten argued that the M5-brane is described by the curve [19]

$$\Lambda_1^{M}t^3+A_1(z)t^2+\cdots+A_2(z)t+\Lambda_2^{M}=0, \qquad (3.7)$$

where $A_i(z)$ is a polynomial of degree M in z, and Λ_i is the dynamical scale of SU$(M)_i$. At constant t, Eq. (3.7) has M roots for z, which correspond to the positions of the M D4-branes at that t. At constant z, Eq. (3.7) has three roots for t, which correspond to the positions of the NS5-branes at that z.

If we write

$$A_i(z)=\prod_{j=1}^{M}(z-a_{i,j}), \qquad (3.8)$$

then the $a_{i,j}$ $(i=1,2;\ j=1,...,M)$ represent the positions of the M D4-branes between the ith and $(i+1)$th NS5-branes. In terms of the SU$(M)\times$SU(M) gauge theory that lives on the branes, for $i=1,2$, the $a_{i,j}$ are the VEVs of the adjoint in the ith SU(M) factor (up to instanton corrections) [19]. The difference in the average of the $a_{i,j}$ on the two sides of an NS5-brane is the bare mass of the corresponding bifundamental hypermultiplet, which we take to be zero,

$$\frac{1}{M}\sum_{j=1}^{M}(a_{1,j}-a_{2,j})=0. \qquad (3.9)$$

The adjoint VEVs are in the Cartan subalgebra of the gauge group, but are not gauge invariant. They transform under the Weyl group (which preserves the Cartan subalgebra), which permutes the VEVs $a_{i,j}$ for fixed i. Instead, the curve should be written in terms of the gauge invariant moduli, which are in general shifted from their classical values by instanton effects. The Seiberg-Witten curve can then be written [18]

$$\Lambda_1^{M}t^3+[P_1(z;u^{(1)}_n,\Lambda_1^{M})+\Lambda_1^{M}]t^2+[P_2(z;u^{(2)}_n,\Lambda_2^{M})+\Lambda_2^{M}]\times t+\Lambda_2^{M}=0, \qquad (3.10)$$

where $P_1(z)$ and $P_2(z)$ are the same function of z, the Coulomb branch moduli $u^{(i)}_n$ defined in Eqs. (3.5) and (3.6), and the dynamical scale of the corresponding SU$(M)_i$ group factor.[3] Equating the VEVs of the two adjoint scalars amounts to setting $P_1(z;u^{(1)}_n,\Lambda_1^{M})=P_2(z;u^{(2)}_n,\Lambda_2^{M})$. In that case, the curve (3.10) factorizes as

$$\Lambda_1^{M}(t+1)\left(t^2+\frac{P(z)}{\Lambda_1^{M}}t+\frac{\Lambda_2^{M}}{\Lambda_1^{M}}\right)=0. \qquad (3.11)$$

If all moduli and dynamical scales are the same for each gauge group factor SU$(M)_i$, this amounts of enforcing a reflection symmetry about the middle NS5-brane. If we fix $x_{10}=0$, the symmetry $x_6\to -x_6$ becomes $t\to 1/t$. If we then equate the dynamical scales Λ_1 and Λ_2, and fix the moduli of both gauge groups to be equal, then the factorization of the curve immediately follows from the $t\to 1/t$ symmetry. Physically, the factorization corresponds to the fact that the hypermultiplet Q can get a VEV from such a point in moduli space, where the Coulomb and Higgs branches intersect, and the Higgs and Coulomb branches decouple [18]. In the M-theory picture giving a VEV to Q corresponds to removing the middle brane, which is at $t=-1$ as we have written

[1]A complete analysis of the moduli space of a more general class of theories which includes this one can be found in [17].

[2]On the reduced Coulomb branch there are additional moduli corresponding to the hypermultiplet VEVs. However, they cannot appear in the Seiberg-Witten curve because of the product structure of the Higgs and Coulomb branches [13].

[3]$P_i(z,u^{(i)}_n,\Lambda_i^{M})$ contains a constant term proportional to Λ_i^{M}. This represents a shift of the maximal moduli $\langle \mathrm{Tr}\phi^{M}\rangle$ and $\langle \mathrm{Tr}\varphi^{M}\rangle$ from their classical values. We have ignored a numerical constant in the definition of the dynamical scale. For more on these subtleties, see for example [17,18].

the curve, in the x_7, x_8, x_9 directions. By rescaling the curve by $t \rightarrow \Lambda_1^2 t$, the dynamics along the reduced Coulomb branch is given by the factor multiplying $(t+1)$, which we recognize to be simply the Seiberg-Witten curve for the diagonal $SU(M)_D$ gauge theory, with dynamical scale $\Lambda_D^{2M} = \Lambda_1^M \Lambda_2^M$. That is, the Seiberg-Witten curve describes the unbroken $U(1)^{M-1}$ gauge theory on the reduced Coulomb branch. Giving a VEV to the hypermultiplet Q corresponds to the removal of the middle brane in the x_7, x_8, x_9 directions, but as mentioned earlier that dynamics is decoupled from the dynamics described by the Seiberg-Witten curve at low energies.

We now see explicitly from the brane construction that the enhancement of SUSY along the reduced Coulomb or Higgs branch is stable to radiative and nonperturbative corrections. The D4-branes break half of the 32 supersymmetries of the type IIA string theory, and the NS5-branes break another half. If the NS5-branes are removed, then the remaining theory at energies below those which probe the NS5-branes is that of the D4-branes on the circle, which has twice as much SUSY. In essence, the theory has factorized into the theory on the parallel NS5-branes with 16 supercharges, and the theory on the D4-branes with a different 16 supercharges. The decoupling is confirmed by the factorization of the Seiberg-Witten curve.

As mentioned in the Introduction, we need to take the number of gauge group factors large in order to be able to probe the size of the torus because otherwise the spectrum differs from the expected KK spectrum at low energies, and the deconstructed theory cannot correspond to a six dimensional theory on a torus. We can understand this from the string theory construction as well by considering the relevant energy scales. The type IIA brane configuration is useful in understanding the gauge theory on that brane configuration at energies less than l_s^{-1}. The (0,2) theory is the theory on a stack of M5-branes with vanishing 11D Planck scale, $l_P^3 = l_s^2 R_{10}$, so we take $l_s \rightarrow 0$. Furthermore, the theory that lives on the wrapped M5-branes is only the (0,2) theory up to scales where they probe the M5-branes that were removed from the torus. At energies much smaller than the inverse sizes of the torus, the (0,2) theory becomes the 4D $\mathcal{N}=4$ theory. We want to arrange the parameters of the theory so that we can probe somewhat higher energies, on the scale of the inverse sizes of the torus, without also probing the NS5-branes. Strings which stretch from a D4-brane to an NS5-brane and back carry energy of order gv, corresponding to the fact that at that energy the gauge theory probes the individual gauge group factors and hypermultiplets Q_i.

The size of the torus is determined by the low lying "electric" spectrum of the gauge theory, which corresponds to the KK modes around the x_6 circle. This fixes $R_6 = N/(2\pi g v)$, so we must have N large in order for the scale of low lying KK modes to be much smaller than the bifundamental hypermultiplet scale gv. Notice that the size of the torus is related to the positions of the NS5-branes and l_s. R_{10} will be related to R_6 by identifying the low energy gauge coupling in terms of R_6/R_{10} via the M5-brane brane construction. Alternatively, we can identify R_{10} via the magnetic spectrum, as in [3], which is interpreted as the KK modes around the other cycle of the torus. For nonvanishing theta parameter in the low energy $SU(M)_D$ gauge group, the low lying states in the large N limit have masses,

$$M_{n,m} = \frac{gv}{N} \left| n + m \left(\frac{\theta}{2\pi} + i \frac{4\pi}{g_D^2} \right) \right| = \frac{1}{2\pi R_6} \left| n + m \left(\frac{\theta}{2\pi} + i \frac{R_6}{R_{10}} \right) \right|, \tag{3.12}$$

where n and m are the electric and magnetic quantum numbers of the Bogomol'nyi-Prasad-Sommerfeld (in the sense of the theory with 16 supercharges) dyon, respectively, and correspond to the KK numbers around the two one-cycles of the torus. Equation (3.12) also determines the complex structure of the torus, $\tau = \theta/(2\pi) + iR_6/R_{10}$, in terms of the gauge theory parameters. Note that the bifundamental VEV v sets the overall scale for the flat metric on the torus, but does not appear in the period τ. This is once again a reflection of the product structure of the Higgs and Coulomb branches where they intersect. The first equality in Eq. (3.12) is the holomorphic extension of the spectrum found in [3], where $v_h = gv$ is the holomorphic VEV. The second equality in Eq. (3.12) follows immediately from the M-theory construction as we will now see.

IV. COMPACTIFICATION ON AN ARBITRARY TORUS: COMPLEX STRUCTURE AND FACTORIZATION

To identify the complex structure of the torus we will closely follow Witten [19]. The M-theory circle in the x_{10} direction has radius R_{10} and the circle in the x_6 direction has radius R_6. The bare gauge coupling of the ith gauge group $SU(M)_i$ is given by the difference in the x_6 positions of the ith and $(i+1)$th NS5-branes [19,26],

$$\frac{4\pi}{g_i^2} = \frac{(x_{i+1}^6 - x_i^6)}{R_{10}}. \tag{4.1}$$

The bare coupling of the diagonal gauge group is given by

$$\frac{4\pi}{g_D^2} = \sum_i \frac{4\pi}{g_i^2} = \frac{R_6}{R_{10}}. \tag{4.2}$$

The theta parameter of the $SU(M)_i$ gauge group factor is given by the difference in the x_{10} positions of the ith and $(i+1)$th NS5-branes [19],

$$\theta_i = \frac{x_{i+1}^{10} - x_i^{10}}{R_{10}}. \tag{4.3}$$

By "going around the circle" in the x_6 direction, namely, by summing Eq. (4.3) over θ_i, we would find

$$x_6 \rightarrow x_6 + 2\pi R_6,$$

$$x_{10} \rightarrow x_{10} + \theta R_{10}. \tag{4.4}$$

By varying θ, the theta parameters in the gauge theory can be chosen arbitrarily, as now $\Sigma_i \theta_i = \theta$. The period of the torus $\tau = \theta/2\pi + iR_6/R_{10}$ is identified with the holomorphic gauge coupling of the diagonal $\mathrm{SU}(M)_D$ gauge theory on the Higgs branch, $\tau = 4\pi i/g_D^2 + \theta/2\pi$. Thus, by varying R_6/R_{10} and θ, or equivalently by varying the gauge couplings and theta parameters of the deconstructed theory, the complex structure of the torus on which the (0,2) theory is compactified can be chosen arbitrarily.

Let us now discuss the factorization of the Seiberg-Witten curve on the intersection of the Higgs and Coulomb branches for the full deconstructed theory. A classical analysis similar to that done in Sec. III indicates that there is a reduced Coulomb branch in this theory along which the Seiberg-Witten curve should factorize. We write the Seiberg-Witten curve as in [19]:

$$z^M - f_1(x,y) z^{M-1} + f_2(x,y) z^{M-2} + \cdots + (-1)^M f_M(x,y) = 0, \tag{4.5}$$

where x and y parametrize the torus on which the M5-brane lives, which is specified by an elliptic curve of the form

$$y^2 = x^3 + c_1 x + c_2, \tag{4.6}$$

for some complex constants c_1 and c_2. These constants, together with the unique nonsingular holomorphic differential $\omega = dx/y$ and a choice of cycles, determine the complex structure of the torus. To be precise, there is a basis of cycles γ_1 and γ_2 on the torus with unit intersection from which the period of the torus can be calculated as

$$\tau = \oint_{\gamma_1} \omega \left(\oint_{\gamma_2} \omega \right)^{-1}. \tag{4.7}$$

The period τ is to be identified with the gauge coupling and θ parameter of the diagonal $\mathrm{SU}(M)_D$ gauge group via $\tau = 4\pi i/g^2 + \theta/2\pi$. The constants c_i in Eq. (4.6) and the cycles γ_i are chosen to give the correct period τ on the moduli space of vacua. There is also a flat metric on the torus which specifies its size.

For generic adjoint VEVs, there are M roots for z, which describe the positions of the D4-branes as before. The functions $f_i(x,y)$ are generically meromorphic functions on the torus which have simple poles. At the pole there would be a singularity in at least one of the roots of z which near the singularity looks like

$$(z - z_0)(s - s_0) = \epsilon, \tag{4.8}$$

where s is a good local coordinate on the torus near the singularity. This would describe the smooth connection of the D4-brane at z_0 to the NS5-brane at s_0. But at points in the moduli space where the Higgs and Coulomb branches meet, the curve factorizes. This happens when the $f_i(x,y)$ are constants. The interpretation of this is as follows. If we assume that $R_6 \gg R_{10}$ then in the neighborhood of an NS5-brane the Seiberg-Witten curve is well described by the coordinates z and $s = \exp(x_6 + ix_{10})$ with the x_6 position of the NS5-brane set to 0, say. (Alternatively we can just focus on an appropriate neighborhood of the NS5-brane for which $|x_6| \ll 2\pi R_6$, and remove the restriction on R_6/R_{10}.) When the Higgs and Coulomb branches intersect there is a symmetry corresponding to reflections about the NS5-brane, which as we saw earlier implies that the curve should factorize near the NS5-branes. This implies that locally the $f_i(x,y)$ in Eq. (4.5) are constants, which by holomorphy implies they are constants over the torus. Then, using the S duality which takes $\tau \to -1/\tau$ and $\tau \to \tau + 1$, we can relax the restriction $R_6 \gg R_{10}$, so that the $f_i(x,y)$ are generically constants on the intersection of the Higgs and Coulomb branches.

In order to completely specify the brane configuration in the case in which it factorizes, the curve (4.5) should instead be written

$$h_0(x,y) z^M - h_1(x,y) z^{M-1} + h_2(x,y) z^{M-2} + \cdots + (-1)^M h_M(x,y) = 0, \tag{4.9}$$

where now the $h_i(x,y)$ are all equal up to overall numerical constants. All of the $f_i(x,y) = h_i(x,y)/h_0(x,y)$ of Eq. (4.5) are then constants. The curve factorizes as

$$h_0(x,y) \prod_{i=1}^{M} (z - z_i) = 0, \tag{4.10}$$

where z_i are the positions of the D4-branes, and the zero of $h_0(x,y)$ specify the positions of the NS5-branes. The singular intersections of the D4-branes and NS5-branes are similar to Eq. (4.8) with $\epsilon \to 0$.

Having understood how the M-theory five-brane configuration contains information about the reduced Coulomb branch that we are interested in, we can now easily understand the emergence of the (0,2) theory from the deconstructed theory. When the VEVs of all the adjoints are equal, the Seiberg-Witten curve describing the brane configuration factorizes into a product of terms reflecting the positions of the flat NS5-branes and the positions of the D4-branes which wrap the x_6 direction. When the NS5-branes are removed symmetrically around the torus, the Seiberg-Witten curve describes the remaining D4-branes, which are really M5-branes wrapped on the x_6-x_{10} torus. If we also take the 11D Planck scale $l_P^3 = l_s^2 R_{10}$ to zero, then up to the presence of the additional NS5-branes this is precisely the $(0,2)_M$ theory on a torus whose complex structure we have identified above. At very low energies the theory on the D4-branes is the 4D $\mathcal{N} = 4$ $\mathrm{SU}(M)$ gauge theory, the dynamics of which is not corrected by the presence of the NS5-branes (as a result of the product structure of the Higgs and Coulomb branches). At higher energies but below the scale gv which probes the positions of the NS5-branes, the spectrum mimics the KK spectrum expected of the $(0,2)_M$ theory on a torus with the same complex structure as that determined by the Seiberg-Witten curve. This provides additional evidence that the deconstructed theory does indeed mimic the (0,2) theory at scales which probe the size of the torus on which the (0,2) theory is compactified.

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